Question 1

If p, q are prime numbers such that $100 < p < q$, then the number of positive integers less than pq which are relative prime to pq is:

- ● $pq - p - q + 1$
- ○ $pq - q + 1$
- ○ $pq - p - q - 1$
- ○ $pq - p - q$
- ○ $pq - 1$
- ○ None of these

Question 2

The number $\left(4^{100} \bmod 6\right)^{100} \bmod 10$ equals

- ○ 3
- ● 6 (All's answer)
- ○ 2
- ○ None of these
- ○ 5
- ○ 1
- ○ 4

Question 3

Consider the set of all 99 positive integers not exceeding 99.

	$\binom{99}{50}$	2^{98}	2^{97}	2^{49}	None of these	2^{50}	$\binom{99}{50}\binom{99}{49}$	$\binom{99}{49}\binom{99}{49}$
How many subsets have an odd number of odd numbers, and an even number of even numbers?	○	○	●	○	○	○	○	○
How many subsets have an odd number of even numbers and an even number of odd numbers?	○	○	●	○	○	○	○	○
How many subsets have 49 elements?	●	○	○	○	○	○	○	○
How many subsets have an odd number of odd numbers?	○	●	○	○	○	○	○	○

Question 4

Consider the following system of congruences:

$$x \equiv 1 \bmod 2$$
$$x \equiv 1 \bmod 5$$
$$x \equiv 7 \bmod 9$$

Indicate the set of all solutions to the above system of congruences.

- ○ None of these
- ○ $\{90 + 7k \mid k \in \mathbb{Z}\}$
- ○ $\{1 + 2k \mid k \in \mathbb{Z}\} \cup \{1 + 5k \mid k \in \mathbb{Z}\} \cup \{7 + 9k \mid k \in \mathbb{Z}\}$
- ○ $\{9 + 90k \mid k \in \mathbb{Z}\}$
- ● $\{61 + 90k \mid k \in \mathbb{Z}\}$ (All's answer)
- ○ $\{7 + 90k \mid k \in \mathbb{Z}\}$

Question 5

Find a greatest common divisor of the polynomials $x^3 - 1$ and $x^3 + 2x^2 + 2x + 1$

- ○ $x + 1$
- ○ 1(Mikkel's answer)
- ○ $x - 1$
- ● $x^2 + x + 1$
- ○ $x^2 + x - 1$
- ○ None of these
- ○ $x^2 - x + 1$
- ○ $x^2 - x - 1$

Question 6

Recall that $\mathbb{N}$ is the set of natural numbers, in other words the set of nonnegative integers. For all $n \in \mathbb{N}$ define $f(n) = \sum_{k=0}^{n} k \cdot k! = 0 \cdot 0! + 1 \cdot ! + ... + n \cdot n!$. It is possible to prove by induction that $f(n) = (n+1)! - 1$ holds for all nonnegative integers n.

By choosing 4 of the following 8 text fragments and putting them in the correct order, a proof by induction for the above statement can be created.

A. To prove the induction step, we assume that $f(n+1) = ((n+1)+1)! - 1$ holds for some $n \in \mathbb{N}$. We will now prove that under this assumption, $f(n) = (n+1)! - 1$

B. To prove the induction step we assume that $f(n) = (n+1)! - 1$ holds for all $n \in \mathbb{N}$. We will now prove that under this assumption, $f(n+1) = ((n+1)+1)! - 1$ holds for all $n \in \mathbb{N}$.

C. To prove the induction step we assume that $f(n) = (n+1)! - 1$ holds for some $n \in \mathbb{N}$. We will now prove that under this assumption, $f(n+1) = ((n+1)+1)! - 1$.

D. The statement now follows from the principle of mathematical induction.

E. We prove the statement by induction. The base case is $n = 1$. For $n = 1$, we see that $f(1) = \sum_{k=0}^{1} k \cdot k! = 0 \cdot 0! + 1 \cdot 1! = 0 \cdot 1 + 1 \cdot 1 = 1$ and also that $(n+1)! - 1 = (1+1)! - 1 = 2! - 1 = 2 - 1 = 1$. This proves the base case.

F. We prove the statement by induction. The base case is $n = 0$. For $n = 0$, we see that $f(0) = \sum_{k=0}^{0} k \cdot k! = 0 \cdot 0! = 0 \cdot 1 = 0$ and also that $(n+1)! - 1 = (0+1)! - 1 = 1! - 1 = 1 - 1 = 0$. This proves the base case.

G. We have that

$$
\begin{aligned}
f(n) &= \sum_{k=0}^{n} k \cdot k! \\
&= \left(\sum_{k=0}^{n+1} k \cdot k!\right) - (n+1)(n+1)! \\
&= f(n+1) - (n+1)(n+1)! \\
&= ((n+1)+1)! - 1 - (n+1)(n+1)! \text{ By the induction hypothesis} \\
&= (n+2)! - (n+1)(n+1)! - 1 \\
&= (n+2)(n+1)! - (n+1)(n+1)! - 1 \\
&= (n+2-(n+1))(n+1)! - 1 \\
&= (n+1)! - 1
\end{aligned}
$$

which is what we wanted to prove. This concludes the induction step.

H. We have that

$$
\begin{aligned}
f(n+1) &= \sum_{k=0}^{n+1} k \cdot k! \\
&= \left(\sum_{k=0}^{n} k \cdot k!\right) + (n+1)(n+1)! \\
&= f(n) + (n+1)(n+1)! \\
&= (n+1)! - 1 + (n+1)(n+1)! \text{ By the induction hypothesis} \\
&= (n+1)(n+1)! + (n+1)! - 1 \\
&= [(n+1)+1](n+1)! - 1 \\
&= (n+2)! - 1 = ((n+1)+1)! - 1
\end{aligned}
$$

which is what we wanted to prove. This concludes the induction step.

	A	B	C	D	E	F	G	H
The first fragment is:	○	○	○	○	○	●	○	○
The second fragment is:	○	○	●	○	○	○	○	○
The third fragment is:	○	○	○	○	○	○	○	●
The fourth fragment is:	○	○	○	●	○	○	○	○

Question 7

For all $n \in \mathbb{N}$ define a_n recursively as follows $a_0 = 2, a_1 = 3, a_n = \begin{cases} a_{n-1}+n \text{ if } n \text{ is even} \\ a_{n-1}+2a_{n-2} \text{ if } n \text{ is odd} \end{cases}$

- ● 37 (All's answer)
- ○ 38
- ○ 35
- ○ 40
- ○ None of these
- ○ 36
- ○ 39

Question 8

We wish to construct a bipartite graph with bipartition (V_1, V_2) such that $|V_1| = 4, |V_2| = 5$ (Note that a bipartite graph has no loops, but it may contain multiple edges.)

	The graph exists without multiple edges.	The graph exists but only if we allow multiple edges.	The graph does not exist, but it will exist if we are allowed to increase one of the degrees in V_1.	The graph does not exist, but it will exist if we are allowed to increase one of the degrees in V_2.	None of these are correct
If the degrees in V_1 are $5, 5, 5, 5$ and the degrees in V_2 are $4, 4, 4, 4, 4$ then	●	○	○	○	○
If the degrees in V_1 are $1, 2, 2, 2$ and the degrees in V_2 are $1, 2, 2, 2, 2$ then	○	○	●	○	○
If the degrees in V_1 are $4, 4, 4, 4$, and the degrees in V_2 are $5, 5, 5, 5, 5$ then	○	○	○	●	○

Question 9

The formula $\binom{m+n}{r} = \sum_{k=a}^{r} \binom{m}{r-k}\binom{n}{k}$ is true for all integers m, n, r satisfying $0 < m < r < n$ if we let the summation start with:

- ○ $a = n$
- ○ $a = r$
- ○ None of these
- ○ $a = n - r$
- ○ $a = m$
- ● $a = r - m$

Question 10

Let $A = \{0, 1, 2, 4\}$, $B = \{0, 1, 3, 5\}$, and $C = \{0, 2, 3, 6\}$. IF the universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7\}$, then which of the following is equal to $((A \cap B) \setminus C) \cup ((B \cap C) \setminus A) \cup ((C \cap A) \setminus B)$

○ $\{0, 1, 2, 3\}$

○ All of these sets

○ $\emptyset$

○ $\{4, 5, 6\}$

○ None of these

○ $\{0, 4, 5, 6\}$

● $\{1, 2, 3\}$

Question 11

Consider the statement "for every positive rational number x there are positive integers a and b such that $x = \frac{a}{b}$ and $\gcd(a, b) = 1$." Which of the following statement in predicate logic is equivalent to this if we let $G(a, b)$ denote the statement " a and b are relatively prime"?

- ○ It is not possible to translate the statement into predicate logic.
- ● $\forall x \in \mathbb{Q}^+ \exists a \in \mathbb{Z}^+ \exists b \in \mathbb{Z}^+ \left(x = \frac{a}{b} \land G(a, b)\right)$
- ○ $\forall x \in \mathbb{Q}^+ \exists a \in \mathbb{Z}^+ \exists b \in \mathbb{Z}^+ \left(G(a, b) \rightarrow x = \frac{a}{b}\right)$
- ○ $\forall x \in \mathbb{Q}^+ \forall a \in \mathbb{Z}^+ \forall b \in \mathbb{Z}^+ \left(x = \frac{a}{b} \land G(a, b)\right)$
- ○ $\forall x \in \mathbb{Q}^+ \exists a \in \mathbb{Z}^+ \exists b \in \mathbb{Z}^+ \left(x = \frac{a}{b} \rightarrow G(a, b)\right)$

Question 12

Given a univerisal set U, which of the following is equal to the set $A \cap \overline{(B \setminus C)}$?

- ○ None of these
- ○ $A \cap B \cap \overline{C}$
- ○ $(A \cup B) \cap \left(A \cup \overline{C}\right)$
- ● $\left(A \cap \overline{B}\right) \cup (A \cap C)$
- ○ $A \cap \overline{B} \cap C$
- ○ $A \cap (C \setminus B)$

Question 13

The empty set is an element of which of the following sets?

- ○ $\{\{\emptyset\}\}$
- ○ None of these
- ○ $\emptyset$
- ○ $\{x \in \mathbb{R} : x < x\}$
- ● $\{\emptyset\}$
- ○ All of these sets

Question 14

Which of the following are tautologies?

- ○ $(p \to q) \lor (\neg q \to \neg p)$
- ○ None of these
- ○ $p \leftrightarrow q$
- ○ All of these
- ● $(\neg p \lor \neg q) \to (\neg(p \land q))$
- ○ $(p \lor q \lor r) \land (p \lor \neg q \lor \neg r)$

Question 15

Consider all possible seatings of $3n$ people around two tables, one with n seats and one with $2n$ seats. Find the number of seatings when

	$\frac{(3n)!}{2(n!)}$	$\frac{(3n)!}{4(n!)}$	$\frac{(3n)!}{4n}$	$2\binom{3n}{n}$	$\frac{(3n)!}{2n^2}$	$\frac{(3n)!}{4n^2}$	$\frac{(3n)!}{8n^2}$	$4\binom{3n}{n}$	None of these
Two seatings are considered the same when each person has the same left and right neighbor.	○	○	○	○	●	○	○	○	○
Two seatings are considered the same when each person has the same neighbors (and we do not care about right or left)	○	○	○	○	○	○	●	○	○

Question 16

For each of the following, determine whether it is surjective/injective, or not a well defined function. Recall that $\mathbb{N}$ is the set of natural numbers, in other words the set of nonnegative integers.

	Not a well defined function	Well defined but neither injective nor surjective	Surjective but not injective	Injective but not surjective	Both injective and surjective
$f : \mathbb{R} \to \mathbb{Z}$ given by $f(x) = 2\lfloor \frac{x}{2} \rfloor$	○	●	○	○	○
$f : \mathbb{R} \to \{x \in \mathbb{R} : x \geq 0\}$ given by $f(x) = \sqrt{x^2}$	○	○	●	○	○
$f : \mathbb{N} \to \mathbb{N}$ given by $f(x) = 2x + 7$	○	○	○	●	○
$f : \mathbb{N} \to \mathbb{N}$ given by $f(x) = x - x^2$	●	○	○	○	○
$f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} x \text{ if } x \in \mathbb{Q} \\ -x \text{ if } x \notin \mathbb{Q} \end{cases}$	○	○	○	○	●

Question 17

Consider all permutations of ABCDE

	12	24	36	48	50	64	None of these
How many contain none of AB, BC, CD	○	○	○	○	○	●	○
How many contain ACE	○	○	○	○	○	○	●
How many contain precisely one of AB, CD	○	○	●	○	○	○	○

Question 18

For each relation on the set of four distinct elements a, b, c, d below, decide which property it has.

	The edges of a hasse diagram	Partial order	Total order but not well ordered	Well-order	None of these	Total order but not partial order	Equivalence relation
$(a, a), (a, b), (a, c)(a, d)$	○	○	○	○	●	○	○
$(a, b), (b, c), (c, d)$	●	○	○	○	○	○	○
$(a, a), (b, b), (c, c),$ $(d, d), (a, d), (d, a)$	○	○	○	○	○	○	●
$(a, a), (b, b), (c, c),$ $(d, d), (d, c)$	○	●	○	○	○	○	○
$(a, a), (b, b), (c, c), (d, d),$ $(a, b), (a, c), (a, d), (b, c),$ $(b, d), (c, d)$	○	○	●	●	○	○	○

Note: AI'er var uenige om den sidste, så har markeret de to svar jeg fik.

Question 19

Which of the following is a recursively defined function for the number of ways to tile an $n \times 2$ board using 2×1 tiles.

- ● $f(0) = 1, f(1) = 1, f(n) = f(n-1) + f(n-2)$ for $n \geq 2$
- ○ $f(0) = 0, f(1) = 1, f(n) = 2f(n-2)$ for $n \geq 2$
- ○ $f(0) = 1, f(1) = 1, f(n) = 2f(n-2)$ for $n \geq 2$
- ○ $f(0) = 1, f(1) = 1, f(n) = f(n-1)f(n-2)$ for $n \geq 2$
- ○ All of these
- ○ $f(0) = 0, f(1) = 1, f(n) = f(n-1) + f(n-2)$ for $n \geq 2$
- ○ None of these

Question 20

Find the coefficient of $x^{15}y^{20}$ in the polynomials below.

	$-\binom{10}{5}2^5$	$\binom{10}{5}2^{15}$	$-\binom{10}{5}2^{15}$	0	None of these	$-\binom{10}{5}2^{10}$	$-\binom{10}{3}2^5$
$(2x^3 - y^4)^{10}$	●	○	○	○	○	○	○
$(1 - 2x^3y^4)^{10}$	●	○	○	○	○	○	○
$(x^3 - 2y^4)^{10}$	●	○	○	○	○	○	○

Question 21

Which of the following is equivalent to the statement " a and b are relatively prime"? The domain for each statement is the set of all positive integers

- ○ None of these three are equivalent to the statement.
- ○ $\neg(\exists c(c \mid a \wedge c \mid b \wedge c > 1))$ is equivalent to the statement, and the other two are not.
- ○ $\forall c(\neg(c \mid a) \vee \neg(c \mid b) \vee (c \leq 1))$ is equivalent to the statement, and the other two are not.
- ● All of these three are equivalent to the statement.
- ○ $\forall c((c \mid a \wedge c \mid b) \rightarrow (c \leq 1))$ is equivalent to the statement, and other two are not.