10.1

11

Let G be a simple graph. Show that the relation R on the set of vertices of G such that uRv if and only if there is an edge associated to $\{u, v\}$ is a symmetric, irreflexive relation on G.

> **Solution:**
>
> - **Symmetric:** If u is connected to v, then v is connected to u
> - **Irreflexive:** Because you are looking at a simple graph, then there will not be a loop.

10.2

5

Can a simple graph exist with 15 vertices each of degree five?

> **Solution:**
>
> No because the total degree would be uneven

43

How many edges does a graph have if its degree sequence is $5, 2, 2, 2, 2, 1$*? Draw such a graph.*

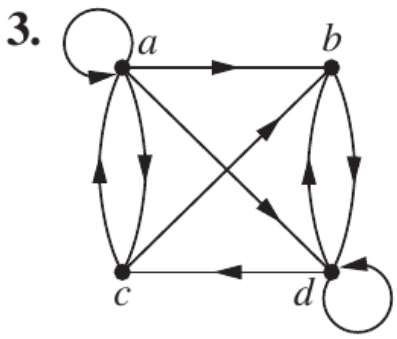
> **Solution:**
>
> $\frac{5+2+2+2+2+1}{2} = 7$

(53)

10.3

(7)

Represent the graph in Exercise 3 with an adjacency matrix.



> **Solution:**

$$\begin{bmatrix} & a & b & c & d \\ a & 1 & 1 & 1 & 1 \\ b & 0 & 0 & 0 & 1 \\ c & 1 & 1 & 0 & 0 \\ d & 0 & 1 & 1 & 1 \end{bmatrix}$$

57

For which integers n is C_n self-complementary?

Solution:

n=5 only

10.4

9

Explain why in the collaboration graph of mathematicians (see Example 3 in Section 10.1) a vertex representing a mathematician is in the same connected component as the vertex representing Paul Erdős if and only if that mathematician has a finite Erdős number.

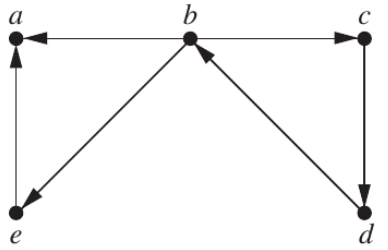
Solution:

Because if a mathematician has a finite Erdos number, then there must be a path of mathematicians who have collaborated connecting them to Erdo.

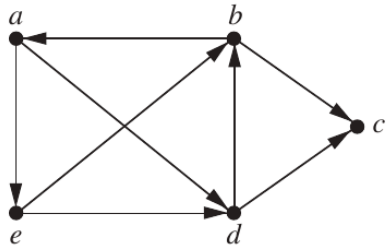
11

Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

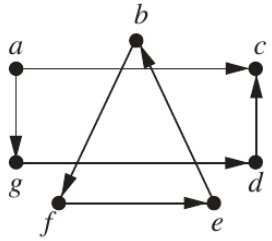
a)



b)



c)



Solution:

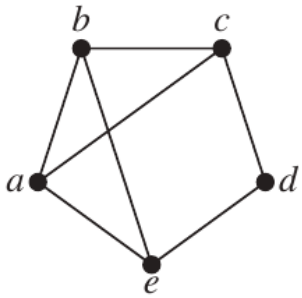
a) No. Because of a. But it is weekly connected
b) No. Because of a-e-b-a. But it is weekly connected.
c) Neither, because of the two subgraphs are not connected

10.5

In Exercises 1–8 determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

(1)

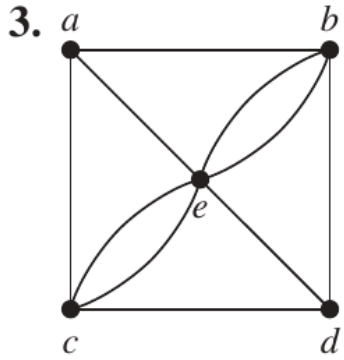
1.



Solution:

Neither

3

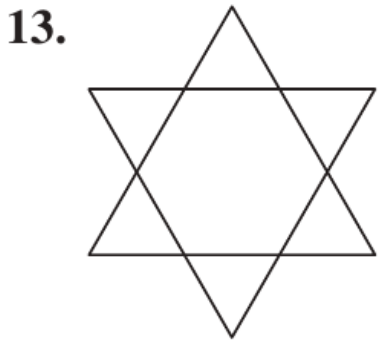


Solution:

No Euler circuit because a has degree 3. It has a path $a, b, e, b, d, e, a, c, e, c, d$

13

In Exercises 13–15 determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture.



Solution:

Yes, draw the triangles first, and then after draw the pentagon in the middle. Also, imagine if every time the lines cross, its a vertex, all the vertices have even edges.