Counting section 6.1 - 6.3

No. of ways toss a coin: 2
No of ways to throw a die: 6

Process 1 in n_1 ways
Process 2 in n_2 ways
⋮
Process k in n_k ways

Process 1 and 2 ... and k can be done in $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ ways

Multiplication rule

No. of ways to first toss a coin and then throw a die: $2 \cdot 6 = 12$

No. of bitstrings of length n written in 2^n ways

No of subset in an n-set S written in 2^n ways because for each element in the set it will either be in or not in the subset

(n-set: set with n size)

No. of functions $f: A \to B$ where $|A| = m$ and $|B| = n$: n^m

No of 1-1 functions $f: A \to B$ where is $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-m+1)$, m

$$= \frac{n!}{(n-m)!}$$

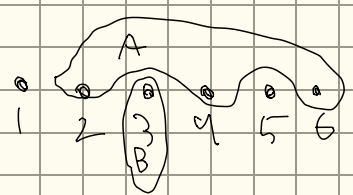
tuples $(5,7), (5,5), (7,5)$

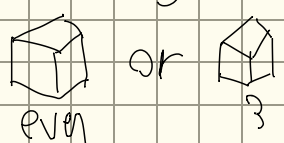
No. of ordered pair (x,y) for an n-set A is n^2

$A \times B = \{(a,b) \mid a \in A, b \in B\}$ is $|A| \cdot |B|$

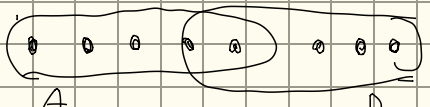
Sum rule

No of ways you can get x or y: ocurrance of x + of y





3 + 1



$|A \cup B| = |A| + |B|$ if $A \cap B = \emptyset$

$|A \cup B| = |A| + |B| - |A \cap B|$

No of bit strings starting with 1 or ending in 00

$2^{n-1} + 2^{n-2} - 2^{n-3}$ if $n \geq 3$

Pigeon hole principal, Dirichlers drawer principal

put m balls into n boxes

if $m > n$ then some box has ≥ 2 balls

some box has $\geq \frac{m}{n}$ balls ← avg balls in boxes

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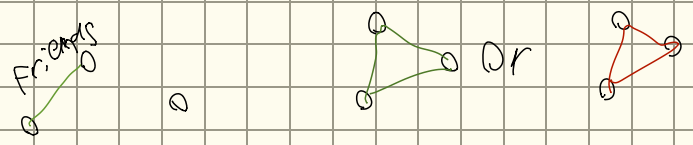
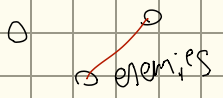
6 people. Any 2 are friends or enemies. Then either there are



3 mutual friends

or 3 mutual enemies

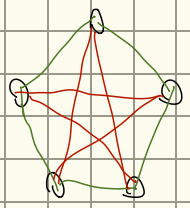


$r(3,3) = 6$ ← Ramsey theory

$r(4,4) = 18$

$r(4,3) = 9$

Not true for 5 people:



No monochromatic triangles

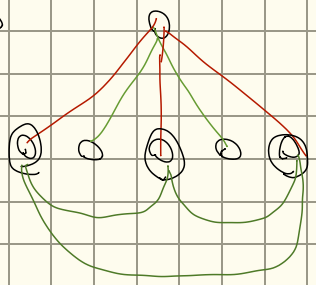
Ramsey theory

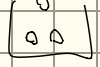
Proof of $r(3,3) \leq 6$ ← pigeonhole

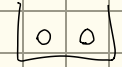
two boxes, every person is a ball

Look at the relations between the red.

Have either one of them are red making the triangle, or all are green making a new triangle







Some box has $\geq \lceil \frac{5}{2} \rceil$ balls

No. of permutations of an n-set

permutation of a set: Amount of ways to organize the set
(for a classroom, the amount of ways the students can sit: $n!$)

n=3: (1 2 3) (1 3 2) (2 3 1) (2 1 3) (3 2 1) (3 1 2)

$P(n,r)$ = the no. of r-permutations of an n-set is
The no. of ways to organize an r-subset of the n-set

$n \cdot (n-1) \cdot (n-2) \cdots (n-(r-1))$

$= \frac{n!}{(n-r)!}$

$C(n,r)$ = the no. of r-combination of an n-set
= the no. of r-subsets of an n-set

P > C
= the no. of ways you can select r-elements from an n-set

$x \cdot r! = P(n,r) = x$

$C(n,r) = \frac{P(n,r)}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \binom{n}{r}$

r or n-r will give same result

The no. of permutations of ABCDEFGH containing ABC

for example GDABCEFH

Think of ABC as 1 letter, so you have 6 elements.
answer is $6!$

$a_1, a_2, a_3, \ldots, a_n$ sequence of numbers

a_2, a_3, a_5, a_{10} subsequence

a_2, a_3, a_5, a_4 not a subsequence because it's not in order

Exercises

Section 6.1: 1, **21, 25, 47,** 57, 69 (67 in the 8th Ed.)
Section 6.2: **7**, 13, **17**, 31, **37**
Section 6.3: **7, 23**, 31, **43**, 47

6.1-21

a) 7: 56, 63, 70, 77, 84, 91, 98

b) 5: 55, 66, 77, 88, 99

c) 1: 77

6.1-25

a) 990

b) 500

c) $q + q + q = 27$
$\times 44 \quad 4\times4 \quad 44\times$
$-444 \quad -444 \quad -444$

6.1-47

$\frac{6!}{6 \cdot 2} = 60$

Fordi du skal kigge forbi dreiningen af hver position og spejlvendinger så $\frac{6!}{6} - \frac{6!}{6 \cdot 2} = 60$

6.2-7

Cause you have 5 numbers, so at least 2 of them will be the same length away from a mult. of 4.

Pigeon hole 4 boxes of the remainders deviding with 4 (0, 1, 2, 3). 5 balls so 1 box has ≥ 2 balls

6.2-17

4

6.2-37

$\left\lceil \frac{377}{38} \right\rceil = 18$

6.3-7

$\frac{9!}{(9-5)!} = 15120$

$n = 9 \quad r = 5$

6.3-27

Permutation for mens pos: 8!
r-Permutation of the womens pos:
$\frac{9!}{(9-5)!}$ 9 dif pos for the 5 women
$8! \cdot \frac{9!}{4!} = 609638400$

6.3-43